SUJET DE THESE G-SCOP 2019*

Titre de la thèse : Graph Structure via Convex Relaxations

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Description du sujet :

One approach for studying classical discrete optimization problems is through the lens of graph structure: What structural properties of a graph make a problem easy? For which restricted graph classes is a problem still hard? Exploiting graph structure to prove bounds and design efficient algorithms has taken many different forms, from connecting graph expansion to the quality of semidefinite programming relaxations to designing combinatorial methods specifically for a family of graphs characterized by a set of forbidden induced subgraphs.

Here, we propose to study algorithms for graph optimization problems on restricted classes through the lens of convex relaxations. For example, in the case of perfect graphs, which have a characterization based on forbidden induced subgraphs, one can use convex relaxations to compute the maximum stable set and chromatic number in polynomial time. However, when studying related---but not perfect---graph classes (for example, graphs with no odd holes or graphs with no induced paths of a particular length) the approach is typically **combinatorial and the utility of convex** relaxations is not well understood. Our goal is to apply the rich toolkit of techniques developed in the last 20 years for designing approximation algorithms based on convex relaxations (iterative rounding, random hyperplane rounding, LP duality) to problems such as graph coloring on undirected graphs and to problems such as finding large induced acyclic subgraphs in directed graphs. In general, these problems are hard to approximate to within any non-trivial factor, and therefore convex relaxations will not be useful. However, assuming some structure on a graph can lead to algorithms with guarantees that are out of reach for general graphs. For example, it has been conjectured that a tournament without a particular forbidden induced subtournament has a large induced acyclic subgraph. Another example is the problem of coloring 3-colorable graphs. While the best known algorithms yield proper colorings with only polynomially many colors, forbidding an induced path of length at most seven allows this problem to be solved in polynomial time.

In summary, our objective is to investigate how convex relaxations and other types of efficiently obtainable representations (for example, sum-of-squares hierarchies of constraints and their duals) can be applied to develop approaches for graph classes whose structure can be used to obtain better bounds and simpler algorithms.

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