SUJET DE THESE G-SCOP 2020

Titre de la thèse : Efficient algorithms for problems with existence proofs, promises or extra information

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Description du sujet :

Sometimes, when studying a discrete optimization problem, we can prove that a solution exists, and only later we discover how to actually find a solution efficiently. An example of this is Spencer's famous result ``Six Standard Deviations Suffice'', which states that a set system on n elements and n sets has discrepancy at most 6 sqrt{n}. Twenty-five years later, several researchers, beginning with Bansal, showed how to find a {-1,1} coloring of the n elements to match this bound, ultimately leading to new, constructive and ultimately simpler proofs of the original result.

Usually this phenomena occurs because, despite being quite an achievement, the proof of existence lacks some insight into or understanding of the problem at hand. However, it is generally thought that a proof of existence implies the existence of an efficient algorithm. In this research project, we propose to study some problems for which the existence of a solution is known, but so far no one has been able to devise an efficient algorithm to find it. More generally, we plan to explore when promises (e.g., a hypergraph being 2-colorable) or auxiliary information (e.g., accesss to correlated instances) can be exploited algorithmically. Conversely, we want to determine what types of promises can make a hard problem easy.

Some classical examples of (hard) open problems for which a solution is known

to exist are related to the following problems :

-- Pigeonhole Equal Sums : find two subsets of a given set

of numbers with equal sums.

-- Number Balancing : find two subsets of a set of numbers, such that the difference of their sums is minimum.

-- Shortest Vector (or lattice problems in general) : find a shortest non-zero vector in a lattice.

-- Diophantine Approximation

A well-known example, where the proof of existence is in fact trivial, is the ``Pigeonhole Equal Sums'' problem. Here, we are given n positive integers whose sum is at most 2^n-2. There are exactly 2^n-1 nonempty subsets of these numbers, and by the pigeonhole principle, at least two of them have equal sums (because the maximum possible sum is 2^n-2). However, it is not known whether we can find two distinct subsets with equal sums efficiently. As a starting point, we may consider the following variations of the problem. What if we make the assumption that the total sum is something much smaller than 2^n-2 (but still exponential)? Or if we look for subsets with sums that almost equal? What if we forbid subsets from having certain sums ? Exploring different conditions/promises that imply the existence of a solution as well as their relation to efficient algorithms may lead to new insights into the problem structure and, in the best case, an efficient algorithms for important variants of the classical problems above.

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