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Titre de la thèse : Depth functions and voting methods; towards a unifying approach

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Description du sujet :

This thesis is part of the CONDORCET project, which aims to renew the analysis of voting rules using robust and interpretable mathematical tools. It explores the writing of voting modes as the result of an optimization problem. Based on statistical depth functions, this approach makes it possible to define, analyze and compare voting methods.

The aim of this work is to formulate voting procedures as optimization problems, in which the outcome of an election corresponds to the maximization of a function of interest. In Aubin et al (2022), the result of a vote is interpreted as the choice of a central, possibly fictitious, voter representing a consensus. This approach requires a rigorous definition of the notion of centrality. It is in this context that depth functions (see Zuo and Serfling, 2000) are a particularly relevant tool. Derived from multivariate statistics, these functions make it possible to quantify the centrality of a point in relation to a data distribution. Transposed to the electoral context, they provide a unifying framework: the winner of an election can be seen as the candidate (or option) that maximizes a depth function defined over the set of preferences expressed by voters. Thus, each voting procedure can be associated with a particular depth function, and reformulated as the solution of an optimization problem based on this function.

Writing voting procedures as the result of depth functions has been proposed by Aubin et al. (2022) in the context of evaluation voting. In this context, voters do not give an order on alternatives, but give a score (evaluation). A first line of research is then to study the link between the properties of depths and voting modes. In particular, depth functions are associated with notions of robustness, i.e. sensitivity to extreme observations, see Liu et al. (2017). Deducing from these properties indicators of the sensitivity to extreme votes of voting procedures is a perspective we would like to explore.

This association between depth functions and voting methods may also make it possible to propose new voting methods. Among the depth functions we'd like to study, we could mention the Monge-Kantorovich function based on optimal transport. Introduced by Chernozhulov et al. (2017), it is associated, like any depth function, with a voting mode whose properties we would like to study.

The majority of voting procedures are based on rankings of alternatives. We refer to Aubin and Rolland (2022) or Felsenthal and Nurmi (2018) for an overview of common voting methods. To transpose previous work into this context, it is therefore necessary to consider depth functions defined on orders or permutations (see the work of Goibert et al (2022)). A second line of research is the analysis of voting modes using rankings thanks to depth functions, and thus writing these voting modes as solutions to optimization problems. For example, one objective is to identify the depth associated with common voting systems, such as Borda, Condorcet, first-past-the-post and second-past-the-post... It is also possible to propose new procedures based on depth functions. By studying the properties of depth functions, we can characterize voting systems.

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